Contents

The Laplace distribution CDF is:

$$F(x) = \begin{cases} \frac{1}{2} + \frac{1}{2}\left(1 - e^{-(x-u)/b}\right), & x > u \\ \frac{1}{2} - \frac{1}{2}\left(1 - e^{(x-u)/b}\right), & x < u \end{cases}$$

It is possible to further reduce this by using the *sgn* (signal) function:

$$F(x) = \frac{1}{2} + \frac{1}{2}sgn(x-u)(1 - e^{-sgn(x-u)(x-u)/b})$$

The quantile function $F^{-1}(x)$ can be found by inverting the cdf:

$$\begin{aligned} \frac{1}{2} + \frac{1}{2}sgn(x-u)(1 - e^{-sgn(x-u)(x-u)/b}) &= y \\ \frac{1}{2}sgn(x-u)(1 - e^{-sgn(x-u)(x-u)/b}) &= y - \frac{1}{2} \\ sgn(x-u)(1 - e^{-sgn(x-u)(x-u)/b}) &= 2y - 1 \\ (1 - e^{-sgn(x-u)(x-u)/b}) &= sgn(x-u)(2y-1) \\ e^{-sgn(x-u)(x-u)/b}) &= 1 - sgn(x-u)(2y-1) \end{aligned}$$

By taking the *log* in both sides:

$$\begin{aligned} -sgn(x-u)(x-u)/b &= log(1 - sgn(x-u)(2y-1)) \\ (x-u) &= -sgn(x-u) \cdot b \cdot log(1 - sgn(x-u)(2y-1)) \\ x &= u - sgn(x-u) \cdot b \cdot log(1 - sgn(x-u)(2y-1)) \end{aligned}$$

So, the $F^{-1}(x)$ function is:

$$F^{-1}(y) = u - sgn(x-u) \cdot b \cdot log(1 - sgn(x-u)(2y-1))$$

This function can still be improved, since the *sgn* still depends on x. Since u is exactly the median of F, i.e., $F(u) = 0.5$, to reproduce the signal of $x - u$ with $y - c$, where c is some constant, we make $c = 0.5$.

So:

$$sign(x - u) = sign(y - 0.5)$$

And the final version of $F^{-1}(y)$ is:

$$F^{-1}(y) = u - sgn(y - 0.5) \cdot b \cdot log(1 - sgn(y - 0.5)(2y - 1))$$