Cheat Sheet

Notation

$\partial_i = \frac{\mathrm{d}}{\mathrm{d}x_i}$

$\boldsymbol{\nabla} = \begin{bmatrix} \partial_1 \\ \partial_2 \\ \partial_3 \end{bmatrix}$

Gradient

Gradient of scalar c

$$\boldsymbol{\nabla} \otimes c = \boldsymbol{\nabla} c^{\mathrm{T}} = \begin{bmatrix} \partial_1 \\ \partial_2 \\ \partial_3 \end{bmatrix} c = \begin{bmatrix} \partial_1 c \\ \partial_2 c \\ \partial_3 c \end{bmatrix}$$

$$(\boldsymbol{\nabla} \otimes c)_i = \partial_i c$$

Gradient of vector $\mathbf{b}$

$$\boldsymbol{\nabla} \otimes \mathbf{b} = \boldsymbol{\nabla} \mathbf{b}^{\mathrm{T}} = \begin{bmatrix} \partial_1 \\ \partial_2 \\ \partial_3 \end{bmatrix} \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} \partial_1 b_1 & \partial_1 b_2 & \partial_1 b_3 \\ \partial_2 b_1 & \partial_2 b_2 & \partial_2 b_3 \\ \partial_3 b_1 & \partial_3 b_2 & \partial_3 b_3 \end{bmatrix}$$

$$(\boldsymbol{\nabla} \otimes \mathbf{b})_{ij} = \partial_i b_j$$

Gradient of matrix A

$$\boldsymbol{\nabla} \otimes \mathrm{A} = \boldsymbol{\nabla} \mathrm{A}^{\mathrm{T}} = \begin{bmatrix} \partial_1 \\ \partial_2 \\ \partial_3 \end{bmatrix} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} \partial_1 \\ \partial_2 \\ \partial_3 \end{bmatrix} \begin{bmatrix} \mathbf{A}_{1k} & \mathbf{A}_{2k} & \mathbf{A}_{3k} \end{bmatrix} = \begin{bmatrix} \partial_1 \mathbf{A}_{1k} & \partial_1 \mathbf{A}_{2k} \\ \partial_2 \mathbf{A}_{1k} & \partial_2 \mathbf{A}_{2k} \\ \partial_3 \mathbf{A}_{1k} & \partial_3 \mathbf{A}_{2k} \end{bmatrix}$$

$$(\boldsymbol{\nabla} \otimes \mathrm{A})_{ijk} = \partial_i A_{jk}$$

Divergence

Divergence of vector $\mathbf{b}$

$$\boldsymbol{\nabla} \cdot \mathbf{b} = \boldsymbol{\nabla}^{\mathrm{T}} \mathbf{b} = \begin{bmatrix} \partial_1 & \partial_2 & \partial_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \partial_1 b_1 + \partial_2 b_2 + \partial_3 b_3$$

$$\boldsymbol{\nabla} \cdot \mathbf{b} = \partial_i b_i$$

Divergence of matrix A

$$\boldsymbol{\nabla} \cdot \mathrm{A} = \boldsymbol{\nabla}^{\mathrm{T}} \mathrm{A} = \begin{bmatrix} \partial_1 & \partial_2 & \partial_3 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} \partial_1 A_{11} + \partial_2 A_{21} + \partial_3 A_{31} \\ \partial_1 A_{12} + \partial_2 A_{22} + \partial_3 A_{32} \\ \partial_1 A_{13} + \partial_2 A_{23} + \partial_3 A_{33} \end{bmatrix}^{\mathrm{T}}$$

$$(\boldsymbol{\nabla} \cdot \mathrm{A})_i = \partial_j A_{ji}$$

Divergence of matrix A^{T}

$$\boldsymbol{\nabla} \cdot \mathrm{A}^{\mathrm{T}} = \boldsymbol{\nabla}^{\mathrm{T}} \mathrm{A}^{\mathrm{T}} = \begin{bmatrix} \partial_1 & \partial_2 & \partial_3 \end{bmatrix} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} \partial_1 A_{11} + \partial_2 A_{12} + \partial_3 A_{13} \\ \partial_1 A_{21} + \partial_2 A_{22} + \partial_3 A_{23} \\ \partial_1 A_{31} + \partial_2 A_{32} + \partial_3 A_{33} \end{bmatrix}^{\mathrm{T}}$$

$$(\boldsymbol{\nabla} \cdot \mathrm{A}^{\mathrm{T}})_i = \partial_j A_{ij}$$